

How can we detect an error without learning the qubit's state?

Noise and quantum error correction

We want to preserve an unknown input:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

We can correct errors without learning the quantum state we are protecting.

- 01 Noise changes the results we are trying to estimate.

- 02 We can encode one qubit's information across three qubits.

- 03 Comparisons reveal an error while preserving the encoded state.

Errors can enter while we store, manipulate or measure qubits.

DURING STORAGE

The qubit interacts with its surroundings.

Energy can be lost; relative phase can drift.

DURING A GATE

The control operation differs from the intended one.

For example, a rotation goes slightly too far.

AT READOUT

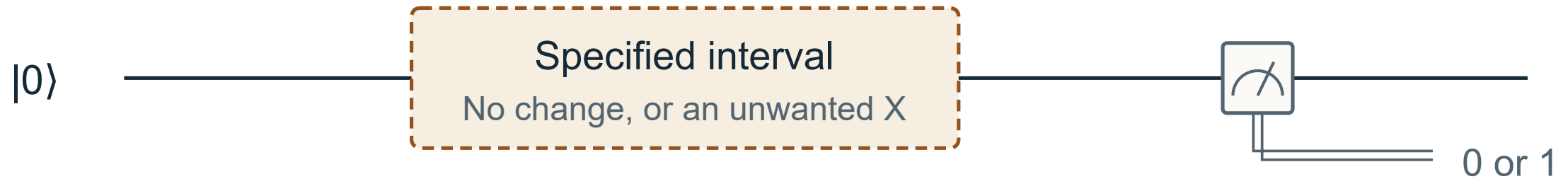
The recorded bit can misreport the outcome.

For example, a 0 is recorded as 1.

Noise can affect the state, the operations and the final records.

A noise model describes how an operation can differ from the intended one.

Our model: an unwanted **X** occurs with probability 10% during this interval.



Modelled event	Probability	Output from input $ 0\rangle$
No error	90%	0
X error	10%	1

A specified fault, location and probability. This 10% is illustrative; it is not a device calibration.

More shots estimate the noisy distribution more precisely.

Ideal prediction: 0% ones

Our noise model: 10% ones

Fresh runs	Recorded ones	Observed frequency
100	13	13%
10,000	1,004	10.04%

Sampling uncertainty shrinks. The noise model still predicts 10% ones.

Illustrative batches from the same model. Actual finite samples vary.

We can reduce physical errors, improve estimates, or protect the quantum information.

SUPPRESSION

Make errors less likely

Remove unnecessary gates and shorten waiting times.

MITIGATION

Improve estimates from counts

For readout errors: test known 0s and 1s, then use the error rates to adjust estimated probabilities.

CORRECTION

Recover the state

For a single bit flip: use extra qubits to locate it, then apply X.

The classical encodings 000 and 111 let us identify a single flipped bit.

Classical bit	Stored pattern	After a flip in the middle
0	000	0 1 0
1	111	1 0 1

With at most one flip, the two agreeing positions identify the original bit.

Only 000 and 111 are valid stored patterns. A single flip creates a recognizable mismatch.

We cannot make identical copies of an arbitrary unknown qubit state.

THREE INDEPENDENT COPIES

$$|\psi\rangle|\psi\rangle|\psi\rangle$$

We cannot copy an arbitrary unknown state this way.

DIRECT DATA READOUT

$$\alpha|0\rangle + \beta|1\rangle$$

Reading each qubit generally destroys the superposition we want to preserve.

The protection has to preserve the unknown amplitudes.

The states 000 and 111 can serve as the two reference states of an encoded qubit.

REFERENCE ZERO

$$|0\rangle \longmapsto |000\rangle$$

REFERENCE ONE

$$|1\rangle \longmapsto |111\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \longmapsto \alpha|000\rangle + \beta|111\rangle.$$

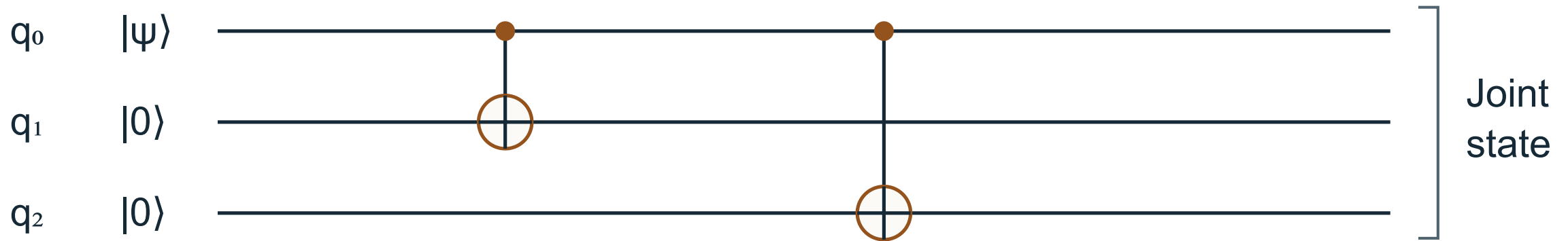
THREE INDEPENDENT COPIES WOULD BE

$$(\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle).$$

For a general superposition, three copies have eight terms; the encoding has two.

One qubit's information is carried by a joint state across three qubits.

Two CNOTs create the encoding from the input and two qubits initially in zero.



Initial $q_0q_1q_2$	After CNOT $q_0 \rightarrow q_1$	After CNOT $q_0 \rightarrow q_2$
$ 000\rangle$	$ 000\rangle$	$ 000\rangle$
$ 100\rangle$	$ 110\rangle$	$ 111\rangle$

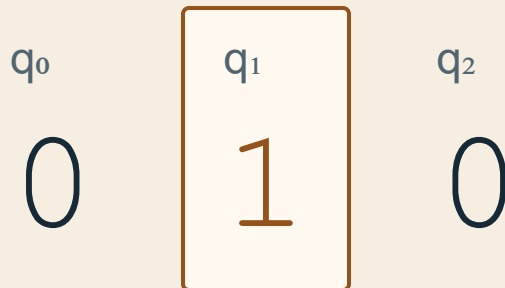
Linearity gives $\alpha|000\rangle + \beta|111\rangle$ for the unknown input.

¹⁰ The input is on q_0 . The three data qubits are written in q_0, q_1, q_2 order.

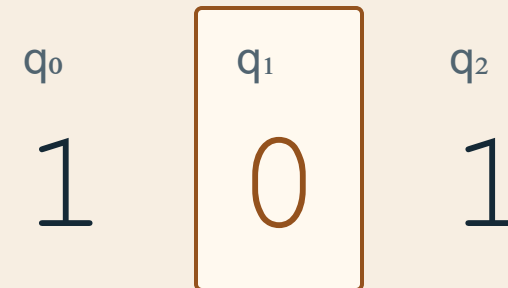
One bit flip creates the same mismatch in both encoded terms.

$$\alpha|000\rangle + \beta|111\rangle \xrightarrow{X \text{ on } q_1} \alpha|010\rangle + \beta|101\rangle.$$

TERM WITH COEFFICIENT A



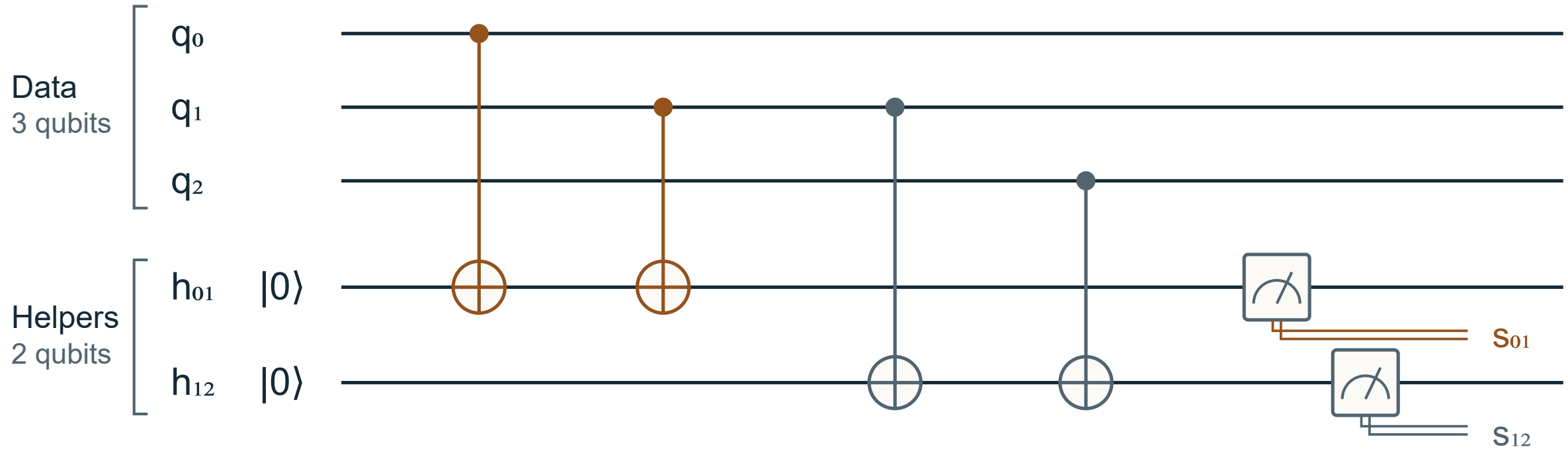
TERM WITH COEFFICIENT B



The middle label differs from both neighbours in each term.

Worked model: at most one X after encoding; encoding, checks and recovery are ideal.

A helper qubit can record whether two data qubits agree.



Do $q_0q_1 = 00$ and 11 give different h_{01} results?

q_0q_1 labels	00	11	01	10
h_{01} readout: s_{01}	0	0	1	1

0 means agreement; 1 means disagreement. Only h_{01} and h_{12} are measured.

These comparisons preserve the superposition because both terms give the same results.

h_{01} compares q_0 with q_1 ; h_{12} compares q_1 with q_2 .

Data term	q_0 compared with q_1	q_1 compared with q_2	Record $s_{01}s_{12}$
$\alpha 010\rangle$	Different	Different	11
$\beta 101\rangle$	Different	Different	11

Can this readout distinguish 010 from 101?

$(\alpha|010\rangle + \beta|101\rangle)$

data $q_0q_1q_2$

$|11\rangle$

helpers $h_{01} h_{12}$

Reading the helpers leaves the data superposition intact.

Two comparison bits locate a single bit flip.

The comparison record is the **syndrome**, written in the order s_{01} , s_{12} .

s_{01} : q_0 with q_1	s_{12} : q_1 with q_2	Error location
0	0	No bit flip
1	0	q_0
1	1	q_1 , the worked example
0	1	q_2

Only the first check reports disagreement. Which qubit could have flipped?

This table assumes at most one X error and ideal checks.

Applying X to the identified qubit restores the encoded input state.

MEASURED SYNDROME $S_{01}S_{12}$

CLASSICAL CHOICE

11 **X on q_1**

$$\alpha|010\rangle + \beta|101\rangle \xrightarrow{X \text{ on } q_1} \alpha|000\rangle + \beta|111\rangle.$$

The original amplitudes survive encoding, checking and recovery.

IF WE UNDO THE TWO ENCODING CNOTS

$$\alpha|000\rangle + \beta|111\rangle \longmapsto (\alpha|0\rangle + \beta|1\rangle)|00\rangle.$$

A phase error can change the encoded state while leaving both comparisons unchanged.

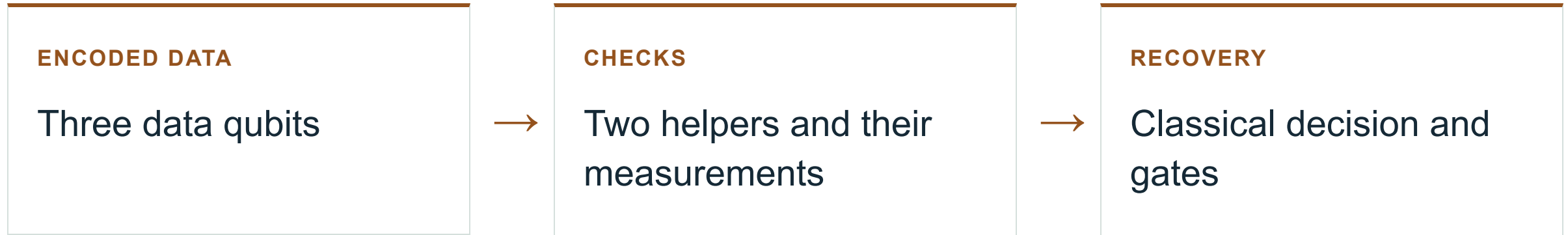
$$\frac{|000\rangle + |111\rangle}{\sqrt{2}} \xrightarrow{Z \text{ on } q_0} \frac{|000\rangle - |111\rangle}{\sqrt{2}}.$$

State	s_{01} from h_{01}	s_{12} from h_{12}
Before Z	0	0
After Z	0	0

The labels still agree. These checks do not detect the changed relative sign.

Two bit flips can also imitate a different single flip. This code does not protect against every quantum error.

Real error correction must also tolerate faults in its checks and corrections.



A wrong comparison record can trigger a wrong correction.

Reliable designs account for faults throughout this process.

Extra qubits and operations are resources to account for. They can introduce errors of their own.

We can correct errors without learning the quantum state we are protecting.

- 01 Noise changes the results we are trying to estimate.
 - 02 We can encode one qubit's information across three qubits.
 - 03 Comparisons reveal an error while preserving the encoded state.
-

The checks identify an allowed error while preserving the unknown amplitudes.

Which correction restores this encoded state?

A single bit flip leaves the data in:

$$\alpha|100\rangle + \beta|011\rangle.$$

01 What do the q_0 – q_1 and q_1 – q_2 comparisons report, in that order?

02 Which qubit receives X to restore the encoding?

03 Why does the helper readout preserve this superposition?

At most one X error; ideal comparisons and recovery. Only h_{01} and h_{12} are measured.