

Qubits, superposition and measurement

Quantum computing manipulates qubit states and uses measurement to obtain classical results.

A qubit is the unit of information we manipulate in quantum computing.

CLASSICAL BIT

0 or 1

QUBIT

Two reference states, 0 and 1,
and their **superpositions**.

PREPARE

Choose a state.



MANIPULATE

Apply operations.



MEASURE

Record a result.

For example: prepare 0 → change the qubit's state to 1.

A quantum state describes the prepared physical system.

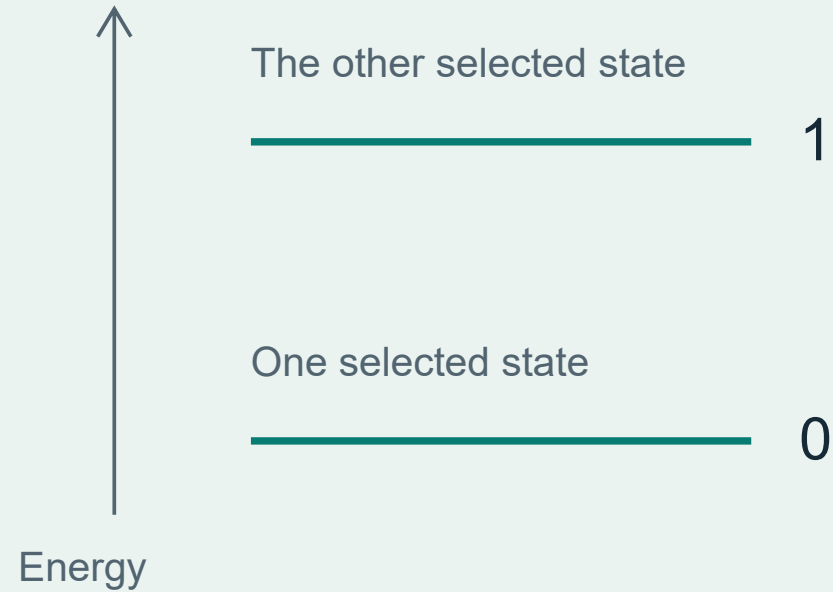
PHYSICAL CARRIER

One atom.

QUBIT

Two selected internal states and their superpositions.

Same physical carrier.
Different prepared states.



Two reference vectors give us coordinates for a qubit's state.

KET ZERO

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

KET ONE

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

These two independent vectors form a basis.

Their linear combinations describe additional states we can prepare.

Superposition allows a qubit to have states beyond zero and one.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

α, β : complex amplitudes. Both are nonzero in a superposition of 0 and 1.

FOR EXAMPLE: PLUS

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

One state of one physical qubit.

Normalization fixes the total squared magnitude at one.

$$|\alpha|^2 + |\beta|^2 = 1 \quad \text{Unit-length state vectors.}$$

START WITH

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Length: $\sqrt{2}$

DIVIDE BY THE LENGTH

$$\rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The plus state.

CHECK

$$\rightarrow \frac{1}{2} + \frac{1}{2} = 1$$

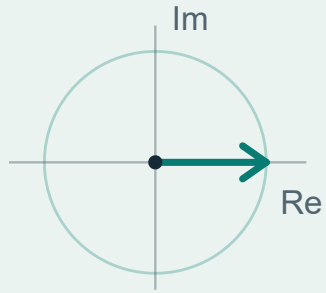
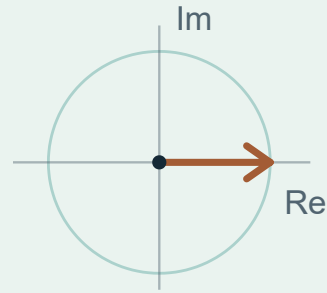
Squared magnitudes.

The amplitudes themselves need not add to one.

Relative phase is part of the state we manipulate.

PLUS

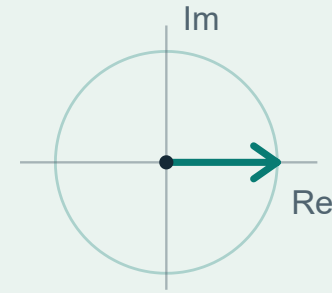
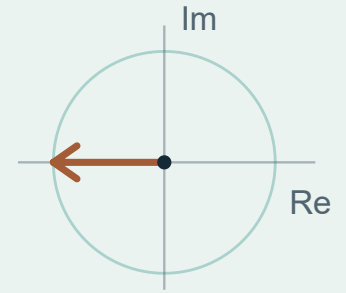
$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Phase of α : 0Phase of β : 0

Relative phase: 0

MINUS

$$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Phase of α : 0Phase of β : π Relative phase: π

Same magnitudes. Different relative phase.

A common phase factor leaves the physical state unchanged.

PLUS

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Multiply both entries by -1 .

SAME PHYSICAL STATE

$$\frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

Both phases change by π . The relative phase stays 0.

This is a change of **global phase**.

The Bloch sphere places these states on its surface.

0 and 1

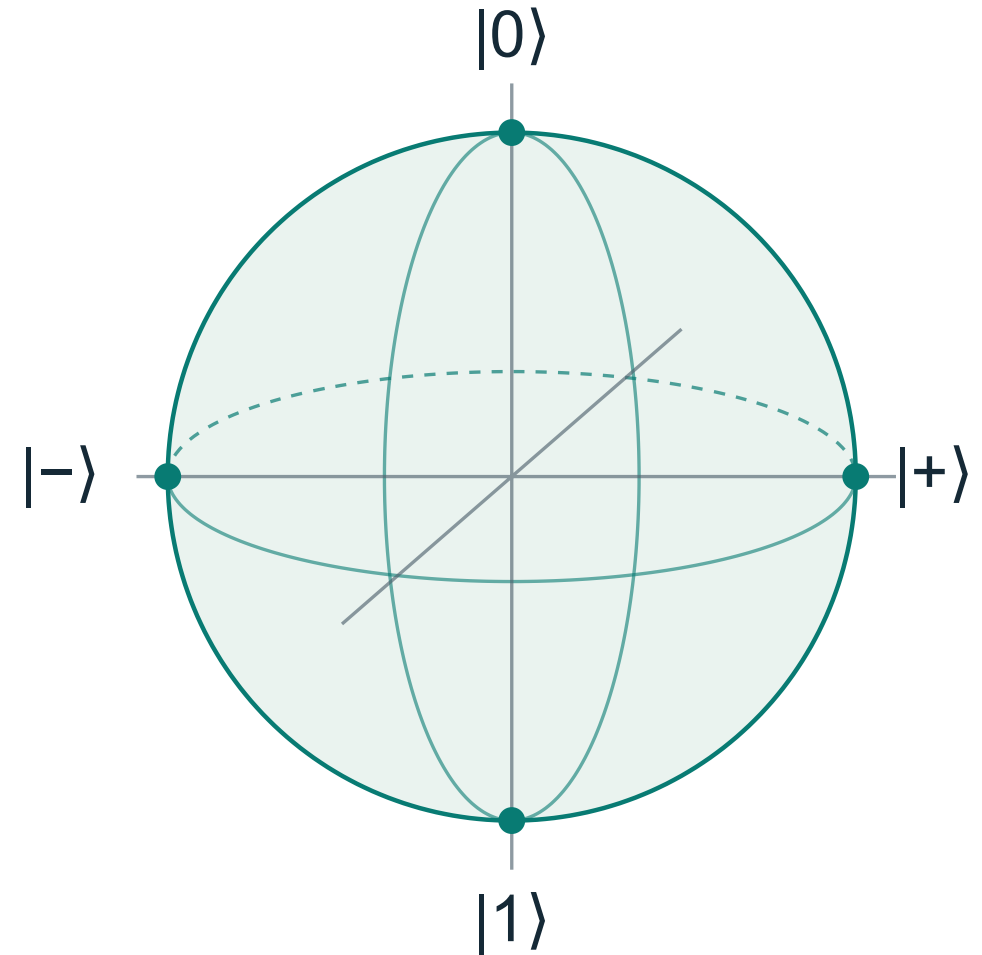
Opposite poles.

Plus and minus

Opposite points on the equator.

Changing only global phase
leaves the point unchanged.

The point describes a state,
not the atom's position in space.



Have we changed the physical state?

Multiply both amplitudes of plus by i .

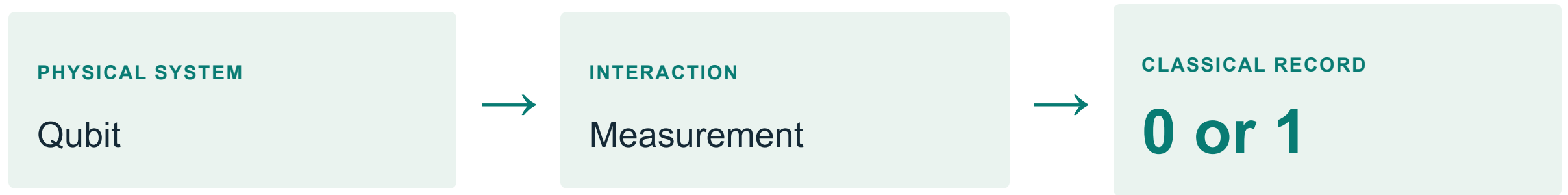
$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ i \end{pmatrix}$$

No. Both phases increase by $\pi/2$.

Their difference stays zero. The physical state is unchanged.

Measurement gives a classical result and can change the qubit's state.

We need information from the physical qubit that we can use as a result.



No measurement can completely determine an arbitrary unknown state from a single qubit, even in principle.

COMPUTATIONAL-BASIS READOUT $|0\rangle \rightarrow \text{record 0}$ $|1\rangle \rightarrow \text{record 1}$

The Born rule connects amplitudes to outcome probabilities.

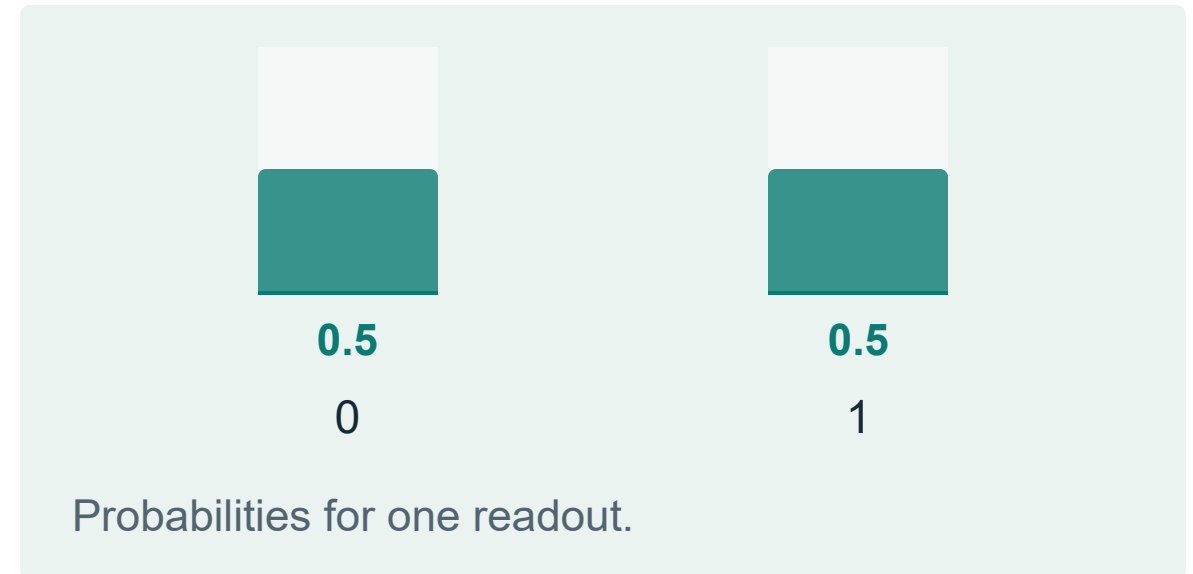
$$P(0) = |\alpha|^2 \quad P(1) = |\beta|^2$$

FOR PLUS

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2}$$

Both outcomes have probability 1/2.



The state determines the probabilities. An individual result can be random.

Measurement can change the qubit's state.

$$|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Ideal computational-basis measurement.



PROBABILITY $\frac{1}{2}$

Classical record

0

State afterwards

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

PROBABILITY $\frac{1}{2}$

Classical record

1

State afterwards

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The qubit remains available. Its state is now the reference state matching the record.

The next calculation uses the state left by measurement.

BEFORE

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Suppose we record 1.



AFTER

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Measure again

**Record 1 with certainty.**

Assume no operation or other change between the ideal measurements.

To repeat the experiment with plus, prepare plus again.

Follow one prepared state through the calculation.

$$|\psi\rangle = \frac{3}{5}|0\rangle + \frac{4i}{5}|1\rangle = \begin{pmatrix} 3/5 \\ 4i/5 \end{pmatrix}$$

One physical qubit. Two nonzero amplitudes.

Does this vector satisfy the normalization condition?

$$\left|\frac{3}{5}\right|^2 + \left|\frac{4i}{5}\right|^2 = \frac{9}{25} + \frac{16}{25} = 1$$

Amplitude phases: 0 and $\pi/2$. **Relative phase:** $\pi/2$.

The Born rule reuses the squared magnitudes.

$$|\psi\rangle = \frac{3}{5}|0\rangle + \frac{4i}{5}|1\rangle$$

COMPUTATIONAL-BASIS MEASUREMENT

$$P(0) = \left| \frac{3}{5} \right|^2 = \frac{9}{25} = 0.36$$

$$P(1) = \left| \frac{4i}{5} \right|^2 = \frac{16}{25} = 0.64$$



The recorded result determines the state afterwards.

BEFORE MEASUREMENT

$$\begin{pmatrix} 3/5 \\ 4i/5 \end{pmatrix}$$

$$P(1) = 0.64$$

Suppose we record 1.



AFTER MEASUREMENT

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The state is $|1\rangle$.

Measure again immediately



$$P(1) = 1$$

Assume no change between the ideal measurements.

Can repeated readouts give the original distribution?

The first measurement recorded 1. The qubit is now in $|1\rangle$.

If we keep measuring this same qubit 100 times, should we expect about 64 ones?

KEEP MEASURING THE SAME QUBIT

No. Every further result is 1.

Its state remains $|1\rangle$, assuming no change between measurements.

PREPARE THE ORIGINAL STATE EACH TIME

$P(1) = 0.64$ on each shot

100 independent shots: 64 ones in expectation. The actual count can differ.

Quantum computing manipulates qubit states and uses measurement to obtain classical results.

- 01 A qubit is the unit of information we manipulate in quantum computing.
- 02 Superposition allows a qubit to have states beyond zero and one.
- 03 Measurement gives a classical result and can change the qubit's state.

What is the probability of measuring 1?

Use a computational-basis measurement.

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Find the amplitude attached to $|1\rangle$.

Take its squared magnitude.

$$P(1) = \left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2} = \boxed{0.50}$$

Find the probability of measuring 1 in each state.

Use a computational-basis measurement.

01 $|0\rangle$

02 $\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$

03 $\frac{\sqrt{3}}{2}|0\rangle + \frac{e^{i\phi}}{2}|1\rangle, \quad \phi \in \mathbb{R}$

Next: [Gates and interference](#) →